GCSE Maths – Geometry and Measures

Sine and Cosine Rules and Area of a Triangle (Higher Only)

Worksheet

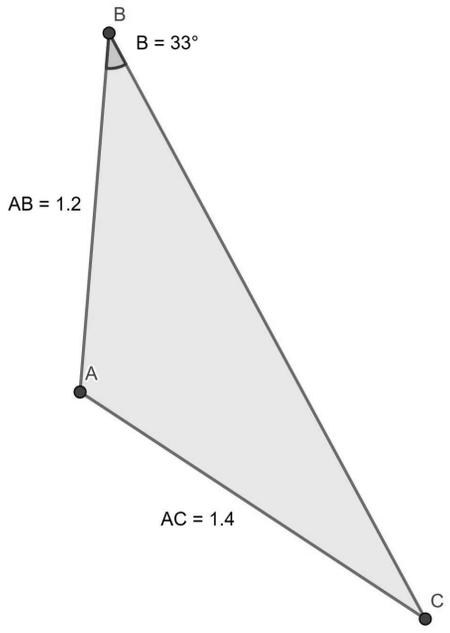
WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions relating to the sine and cosine rules and area of a triangle. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

The triangle ABC is shown below.
Calculate the size of angle C.



Step 1: Identify which rule needs to be used.

Since we know two side lengths and one of their opposite angles, we use the sine rule.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Step 2: Work out which form of the chosen rule to use.

As we are looking to work out the angle, $\sin C$ *needs to be the numerator. We will use this form of the sine rule:*

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

Step 3: Substitute in the values we already know, rearrange the equation to make the unknown term the subject, and calculate the unknown term.

$$\frac{\sin 33}{1.4} = \frac{\sin C}{1.2}$$

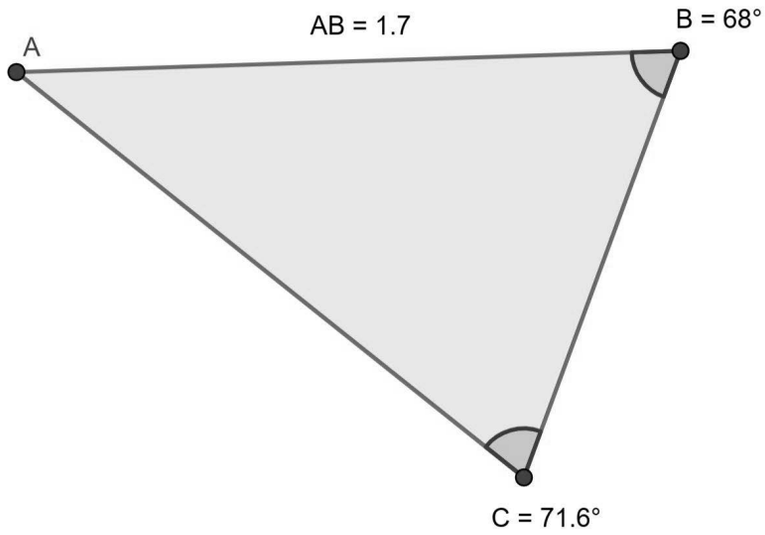
$$\sin C = \frac{\sin 33}{1.4} \times 1.2$$

$$\sin C = 0.46683\ldots$$

$$\sin^{-1} 0.46683\ldots = 27.8268\ldots° = \mathbf{27.83°}$$

Guided Example

Triangle ABC is shown below.
Calculate the length of side AC.



Step 1: Identify which rule needs to be used.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Step 2: Work out which form of the chosen rule to use.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Step 3: Substitute in the values we already know, rearrange the equation to make the unknown term the subject, and calculate the unknown term.

$$\frac{1.7}{\sin 71.6} = \frac{b}{\sin 68} \quad \times \sin(68)$$

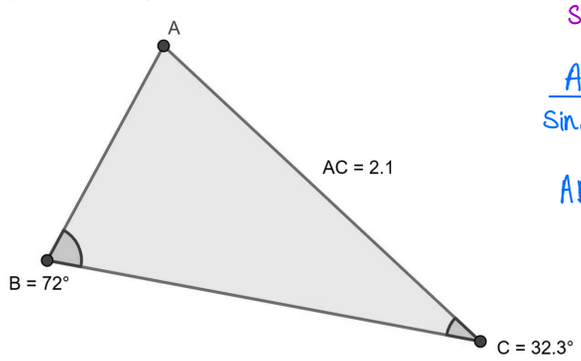
$$AC = \frac{1.7 \sin 68}{\sin 71.6} = 1.661\ldots$$

$AC = 1.66$ units (3sf)

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Calculate the following using the triangles shown below:

a) The length of *AB*



$\frac{a}{\sin A} = \frac{b}{\sin B}$

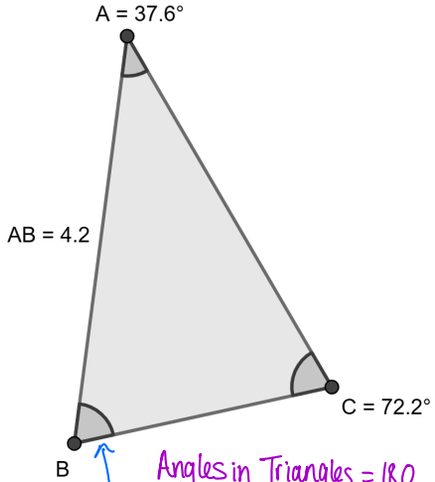
$\frac{AB}{\sin 32.3} = \frac{2.1}{\sin 72}$

$AB = \frac{2.1 \sin 32.3}{\sin 72}$

$= 1.179...$

$= 1.18$ units (3sf)

b) The length of *AC*



$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{AC}{\sin 70.2} = \frac{4.2}{\sin 72.2}$

$AC = \frac{4.2 \sin 70.2}{\sin 72.2}$

$= 4.150...$

$= 4.15$ units (3sf)

Angles in Triangles = 180

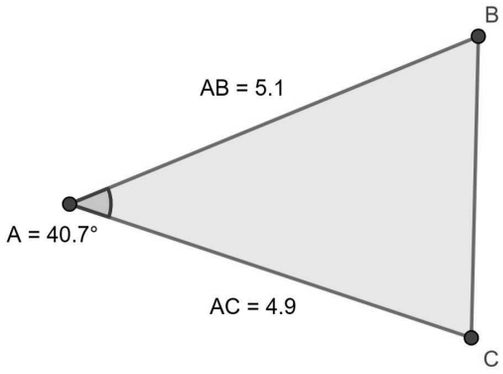
$180 - 72.2 - 37.6$

$= 70.2$

Section B

Worked Example

Consider the triangle below.
Calculate the length of *BC*.



Step 1: Identify which rule needs to be used.

Since we know two side lengths and the angle between them, we use the cosine rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Step 2: Work out which form of the chosen rule to use.

We are calculating the length of BC.
In the cosine equation:

$$a = BC$$
$$b = AB$$
$$c = AC$$

Therefore, we use the form:

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos A$$

Step 3: Substitute in the values we already know, rearrange the equation to make the unknown term the subject (if necessary), and calculate the unknown term.

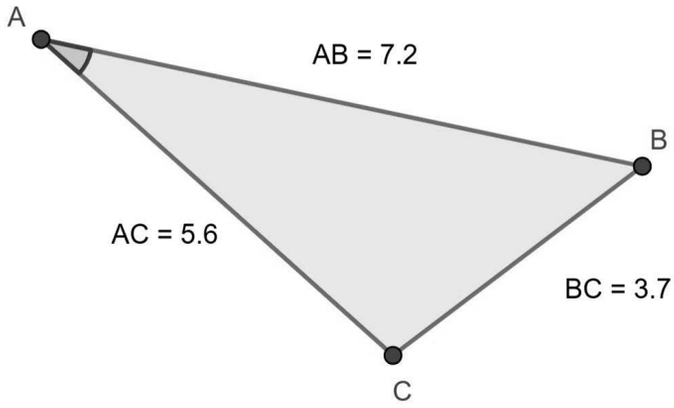
$$BC^2 = 4.9^2 + 5.1^2 - 2 \times 4.9 \times 5.1 \times \cos 40.7$$

$$BC^2 = 12.128\ldots$$

$$BC = \sqrt{12.128\ldots} = 3.482\ldots = \mathbf{3.48}$$

Guided Example

Consider triangle ABC below.
Calculate the size of angle *A*.



Step 1: Identify which rule needs to be used.

Cosine rule

Step 2: Work out which form of the chosen rule to use.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Step 3: Substitute in the values we already know, rearrange the equation to make the unknown term the subject (if necessary), and calculate the unknown term.

$$\cos A = \frac{5.6^2 + 7.2^2 - 3.7^2}{2 \times 7.2 \times 5.6}$$

$$\cos A = \frac{69.51}{80.64}$$

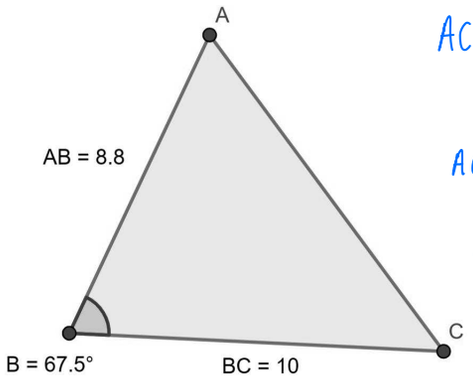
$$A = \cos^{-1}\left(\frac{69.51}{80.64}\right)$$

$$\angle A = 30.5^\circ \quad (3sf)$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. Calculate the following using the triangles shown below:

a) The length of *AC*



$a^2 = b^2 + c^2 - 2bc\cos A$

$AC^2 = 8.8^2 + 10^2 - 2(8.8)(10)\cos 67.5$

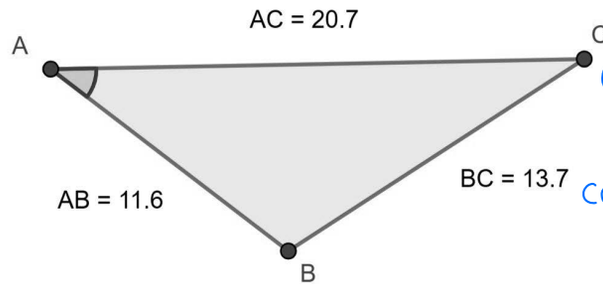
$= 177.44 - 176\cos 67.5$

$AC^2 = 110.0877...$

$AC = 10.49$

$= 10.5$ units (3sf)

b) The size of angle *A*



$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \frac{20.7^2 + 11.6^2 - 13.7^2}{2 \times 20.7 \times 11.6}$

$\cos A = \frac{375.36}{480.24}$

$A = \cos^{-1}\left(\frac{375.36}{480.24}\right)$

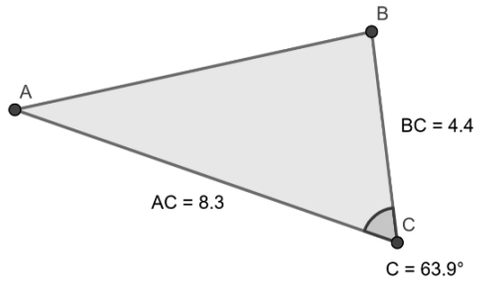
$= 38.59...$

$\angle A = 38.6°$ (3sf)

Section C

Worked Example

Calculate the area of the triangle *ABC*.



Step 1: Identify the available information and decide what form of the area formula we need to use, rearranging if necessary.

$$Area = \frac{1}{2}ab\sin C$$

Since we know the length of side BC (which is 'a' in the formula), the length of side AC (which is 'b' in the formula) and the size of angle C, we will use the standard area formula:

$$Area = \frac{1}{2}ab\sin C = \frac{1}{2}(AC)(BC)\sin C$$

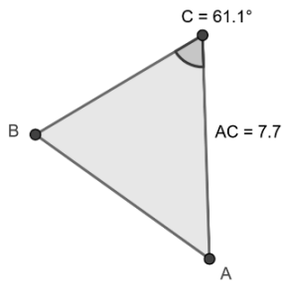
Step 2: Substitute the values that we already know in and calculate the unknown value.

$$Area = \frac{1}{2} \times 4.4 \times 8.3 \times \sin 63.9$$
$$Area = 16.397\ldots = \mathbf{16.40\ units^2}\ \mathbf{(2\ d.p.)}$$

Guided Example

The area of this triangle is 22.7 cm².

Calculate the length of *BC*.



Step 1: Work out what information is available, and what form of the area formula we need to use, rearranging if necessary.

$$Area = \frac{1}{2}ab\sin C$$

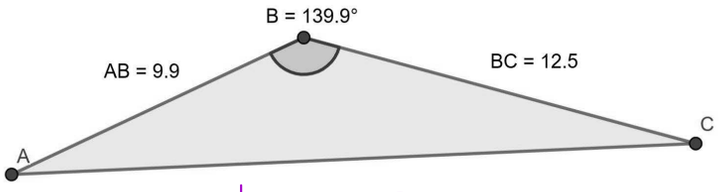
Step 2: Substitute the values that we already know in and calculate the unknown value.

$$22.7 = \frac{1}{2} \times 7.7 \times b \times \sin 61.1$$
(×2) (÷ 7.7 sin 61.1)
$$b = 6.734\ldots$$
$$BC = 6.73\ \text{units}\ (3\text{sf})$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

3. Calculate the following using the triangles shown below:

a) The area of this triangle



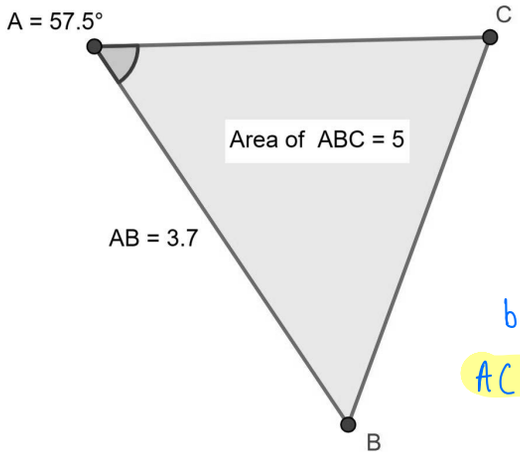
$\text{Area} = \frac{1}{2}ab\sin C$

$\text{Area} = \frac{1}{2} \times 12.5 \times 9.9 \times \sin 139.9$

$= 39.85...$

$\text{Area} = 39.9 \text{ units}^2$ (3sf)

b) The length of *AC*



$\text{Area} = \frac{1}{2}ab\sin C$

$5 = \frac{1}{2} \times 3.7 \times b \times \sin 57.5$

$\times 2$

$\div 3.7\sin(57.5)$

$b = 3.204...$

$AC = 3.20 \text{ units}$ (3sf)